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Constatant les problèmes que posent la formation et le perfectionnement des enseignants de mathématiques, ce document soumet cinq interrogations susceptibles d'éclairer la question: que signifient l'apprentissage et l'enseignement pour les étudiants et pour les enseignants? (chapitre 1); quelles sont les conséquences significatives de la formation actuelle des enseignants? (chapitre 2); quels sont les résultats de la recherche à propos des réformes à mettre de l'avant dans la formation des enseignants? (chapitre 3); comment pouvons-nous favoriser des changements dans les attitudes et les comportements routiniers chez les enseignants? (chapitre 4); quelles sont les options qui s'offrent à nous parmi les changements souhaitables dans la formation des enseignants de mathématiques? (chapitre 5).

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CENTRE INTERDISCIPLINAIRE DE RECHERCHE SUR
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TEACHERS PRE AND IN-SERVICE EDUCATION FOR MATHEMATICS TEACHING

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With the dissolution of the former teacher training colleges or academies and the related integration of Primary teachers' education into the universities in many countries new problems arose. In mathematics particularly the approach to more (and "pure") mathematics under the dominance of mathematical faculties had a high price: the distancing from school reality and classroom problems and a negative selection of future teachers. In Germany (West) the problem thickened across two decades without getting noticed: the annual increase of our population dropped from about 1 million (1960-) down to 550 000 (1975-1980), slightly increasing now, and no young teachers were hired by the state. Consequently the number of student teachers at the universities was near to a collapse and vacant professorships were cut off. The primary schools lived quietly, not challenged by incoming young teachers with new ideas. Many school staffs arrive at an average age of over 50 now. But also the schools got accustomed to arrange for the necessary inservice education by themselves. Thus the rift between universities and schools has grown reasonably.

Now things begin to change dramatically. Within the next 10 years about two thirds of our primary teachers will retire and they become replaced by a majority of young teachers, who at their universities had less practices and school contacts, had less integrated seminars and lectures in many more subject matter areas, and had more 'higher' mathematics. Aside of that the childrens' learning prerequisites have become much more diverse, due to guestworkers, refugees, and immigrants from many countries. The teacher's tasks have become much more complicated than they were a few decades ago.

Though the problems upcoming seem to be at hand, there is no related discussion of adequate actions. In other countries things may function smoother, but the fundamental problems with teacher education will be more or less the same. What is going wrong with this professional preparation? Which new demands arise for inservice support and refresher courses?

Clearly, there are no definite or simple answers. What is discussed in the following, therefore, cannot be more than offers or possible options for future development and small scale reform experiments, open to interested educators who might make sense from it. The more it becomes necessary to lay open the theoretical backgrounds, the perspective, from which potential solutions and alternatives are developed. The following remarks are grouped around five questions:

What can *learning and teaching* mean, for students as well as for teachers? (ch. 1)
What are *relevant effects* from present teacher education? (ch. 2)
What does *research* say *about necessary reforms* of teacher education? (ch. 3)
How could we promote necessary *changes of teachers basal orientations* and routines? (chapter 4) and
What are *options for the necessary changes* in teacher education? (chapter 5)

1. What does learning and teaching mean?

> The "dominant model of teacher education ...: the university provides the theory, the methods, and skills; schools provide the classroom, curriculum, and students; and the student teacher provides the individual effort... This training model, however, ignores the role of the social and political context of teacher education while emphasizing the individual's effort. Here, the social problem of becoming a teacher is reduced to an individual struggle." Britzmann (1986, p. 442).

From a social interactionist perspective (Bauersfeld, 1993b) the key problem of all organized learning is: What are "the conditions for the opportunity that" the learner will change her/his ways, means, and routines of interpreting and acting? Following a famous phrase from Kant this definition points at the four key issues:

- The **active, constructive part** of the learner—the change has to be processed by the learner, not only by the teacher; the learner is the main acting person in the process, since it is his/her own restructuring.
- The **impact of the learner's socialization,** of what has been learned before—the learner permanently acts through reactivating expectations, routines and means, formed already in situations she/he relates to the present situation; there is no "tabula rasa".
- The **character of chance** in teaching—teaching seen as seriously organizing learning processes can provide for opportunities for learning only, there is no reliable "transmitting" or "passing over" of knowledge; the teacher can provide opportunities for subjective restructurings only.
- The **social dimension** of the teaching-learning process—*the notion of "conditions" is* meant to generally include all relevant dimensions, not only subject matter, psychological factors, teaching methods, but also the social interaction and cooperation within the institution, the culture of the classroom, the cultures in the background etc.

Speaking about learning usually will evoke the association with students. But the characteristics of learning apply as well to the teacher's learning. We do know much more about students' learning than we know about the conditions of teachers' professional learning. Regulations and models for initial teacher education particularly underestimate—if not neglect—power and impact of the student teachers' own classroom experiences as a student, the teacher's socialization as a student, which covers more than a dozen years regularly (see Pinner & Shuard, 1985). And these models usually overestimate the socialization effects of an academic training in pure mathematics. From this perspective teacher education is the *re-* education of former school students rather than an introduction into a new profession.

The consequences for a reform of the education of primary mathematics teachers drawn here make use of the findings from research projects at our institute and draw from similar research work in many other classrooms and countries.

2. Relevant Effects from Common Teacher Education

> "Psychologists and educators are far short of charting the full extent of childrens' capacity for understanding." Michael Siegal (1991, p.133)

For the understanding of learning and teaching mathematics the preceding analyses support a model of *participating in a culture* rather than a model of transmitting knowledge, or of introducing students into a body of objective knowledge. Participating in the processes of a mathematics classroom is participating in a culture of using mathematics, or better: a culture of mathematizing as a practice. The many skills, which an observer can identify and will take as the main performance of the culture, form the procedural surface only. These are the bricks for the building, but the design for the house of mathematizing is processed on another level. As it is with cultures, the core of what is learned through participation is *when* to do what and *how* to do it. Knowledge (in a narrow sense) will be for nothing once the user cannot identify the adequateness of a situation for use. Knowledge, also, will not be of much help, if the learner is unable to flexibly relate and transform the necessary elements of knowing into his/her actual situation. This is to say, the core effects as emerging from the participation in the culture of a mathematics classroom will appear on the metalevel mainly and are 'learned' indirectly.

"Learning", by the way, is used as a wide concept here. Particularly we do not, limit this issue, as activity theory does (see Bauersfeld, 1992), to the person's conscious processing. We include the subconscious development of regularities as they emerge across the person's active participation in the specific (classroom) culture.

What has been described for the student here is valid as well for the teacher and his/her orientation for action. Teacher education at universities and colleges functions like other institutional subcultures, and this independently from whether the institution reflects such effects or not. In all countries the curricula for educating mathematics teachers excessively emphasize mathematical understanding, knowledge, and skills. The usual academic ways of mathematical lecturing and exposing mathematical knowledge are quite different from how mathematics can be taught on the elementary level in particular. Consequently, the mathematical habits student teachers' are developing during their preparation at universities will not have much in common with the school mathematical habits a teacher will need in school practice. And what is taught at universities about the realities and conditions of teaching also is more of the knowledge and skills type rather than an indirectly formed product from participating in an adequate processing of it,

where "adequate" means near enough to the teacher's later professional situation in school practice.

Obviously and surprisingly, nobody discusses the question of whether these ways provide for an adequate "enculturation[1]" for the teacher's role in the classroom. A very recent review of fourty learning-to-teach studies has led Kagan (1992) to the summary:

— the studies testify "the stability and inflexibility of [the student teachers'] prior beliefs and images" (p. 140), moreover

— "these personal beliefs and images generally remain unchanged by a preservice program and follow candidates into classroom practica and student teaching" (p. 142), and, consequently,

— "quickly disillusioned and possessing inadequate procedural knowledge, novice teachers tend to grow increasingly authoritarian and custodial" (p. 145)[2].

The neglect of the crucial fit between the teacher's habitus as is emerging from the university education versus the teacher's professional habitus as is required in school practice gives away the chance to counterbalance the student teacher's own socialization during the 12-13 years at school as a student. Once the young teacher enters school again, it will be too late. Under the pressure of the everyday duties and difficulties the chances for theoretically guided change and reform will be small. Winner in conflicting situations then is the reliably rooted and functioning habitus from the own experiences as a student, which readily serves the old 'solutions' and which, through these regressions, promotes the reproduction of the old school.

Which kind of attitudes and which kind of meta-knowledge will future mathematics teachers need, and which ones are produced across the usual ways of educating them? The following two chapters will discuss this more detailed.

3. Recommendations from Related Research

> "You don't learn speaking a language by learning a theory that can be formulated explicitly, you learn it by participating in a certain practice - you pick it up. Now picking up a language enables you to do two things. It enables you to understand and to use certain regularities though without knowing what they are ..."
>
> Feyerabend (1991, 116/117; emphasis in the original)

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1 The notion of "enculturation" is used here in a metaphorical way. I do not use it elsewhere because of the related connotation of becoming en-culturated into something like an already existing and objectively given culture. The culture of a classrom is an emergent social system and not pregiven.

2 The recommendations of these authors for a "new model of teacher development" follow Berliner's schema and favorize an atheoretical approach to "standardized routines that integrate management and instruction" under the headline "procedural, not theoretical knowledge" (p. 162). "One might begin to question whether formal theory is relevant to teachers at any point in their professional development" (p. 163). No wonder, that the directly following commenting contribution from Grossman (1992) contradicts heavily.

If the reality of the mathematics classroom is understood as permanently developing and interactively constituting, then simple cause-and-effect rules cannot provide for a sufficient orientation of the teacher's permanent need for decisions. What characterizes effective teachers—aside of the bunch of subject matter knowledge they are exposed to usually—is a certain "habitus". Originally Pierre Bourdieu has defined habitus as a "generative mechanism" enabling a person to act adequately as a member of her/his social class, even in situations never experienced before. Transferred to the teacher's role in the classroom culture, the concept of habitus exactly describes the individual background for the actual genesis of the subjective "definition of the situation" and of the related issues of the "when and how" to do something adequately or in an acceptable manner.

Putnam, Lampert & Peterson (1990) have summed up an extended review on teaching and learning mathematics in elementary schools by saying, that "three aspects of knowing mathematics" are of particular importance:

"If teaching and learning in school are to lead to genuine academic accomplishments in mathematics, it is reasonable to assert that

- they must be built around the corpus of "big ideas" that are fundamental to mathematics and mathematical thinking" (p. 136),
- "there are conventions of mathematics that must essentially be presented to students" (like digits, %-symbol, x- and y-axes etc.), and thirdly
- "mathematics practice in schools need to be personal sense making" (p. 137),

and furthermore "two features of instruction":

- "talking about mathematics", and
- "considering the kinds of mathematical activities in classrooms" (p. 138).

Of special interest is the idea of "personal sense making". Aiming at the student teacher's own active "work to integrate new information" and "to provide settings in which students' attempts to make sense of new ideas are valued and explored" (p. 137), this seems to be very near to the notion of culture, as is preferred here. Interesting, also, is the stressed importance of "talking about mathematics". This again is close to our considerations, when the authors state: "Verbal discourse ... constitutes an important means of reveiling individual knowledge" and *"much of what is learned about mathematics is implicit."* (italics added) The teacher, therefore, "will need to talk with students, challenging these with counterexamples rather than with judgments about the wrongness of their answers" (p. 138).

Reviewing related research work from about 1985-1990, Anne Reynolds (1992) has arrived at the following synthesis of expectations for beginning teachers. Sharper and clearer she demands: "Beginning teachers should enter the first year of teaching with:

- Knowledge of the subject matter they will teach;
- The disposition to find out about their students and school, and the ethnographic and analytic skills to do so;
- Knowledge of strategies, techniques, and tools for creating and sustaining a learning community, and the skills and abilities to employ these strategies, techniques, and tools;
- Knowledge of pedagogy appropriate for the content area they will teach; and
- The disposition to reflect on their own actions and students' responses in order to improve their teaching, and the strategies and tools for doing so."

Reynolds (1992, p. 25/26)

In the next chapter these useful recommendations are backed-up by a more integrated theoretical framework.

4. How to promote Changes of Teachers' Habitus?

> "The principal "motive" in language acquisition is the better regulation of these underlying social cultural processes ... that are shared by prelinguistic and linguistic communication." Bruner (1983, p. 128)

The fundamental difficulty and trickiness with the genesis of habitus itself is: There is no way for a transmitting of desired characteristics by lecturing about or teaching these directly. But, exposing students to lectures inevitably contributes to the forming of an habitus, as does every participation in the social interaction of institutions or stable groups. And this habitus, emerging from the student teachers' experiencing lectures and seminars, can be blamed mostly for the reproduction of the well-known habits of lecturing and talking into which have been documented so often in regular classrooms (see Hoetker & Ahlbrand, 1969).

If the formation of habitus is a product of enculturation, then we will have to organize the **initial teacher education like a culture which itself lives the desired characteristics.** The notion of "living" is chosen to point at the necessity of active participation in a *culture of cooperative mathematizing*. This notion is to design a process in which the student *and* the relevant others do it, and, right after such activities, discuss and reflect their experiences and develop these into taken-as-shared issues. Clearly, *after* having done it *together* rather than through replacing the original experience by only talking about experiences of others or by lecturing about the expected outcomes. These considerations have lead us to support an alternative practice of teaching mathematics education as well as of teaching mathematics itself to student teachers. Student teachers have to have special lectures and seminars in mathematics.

Related to instructions about the "what," the subject matter mathematics in particular, the many problem solving attempts (see Polya's excellent books) may be taken as helpful starting

points. Following the constructivist principle, the learning develops from subjective constructions and arises from the joint negotiating of the social conventions and norms of mathematizing rather than from 'reading' or 'discovering' objectively 'given' truths or external structures. Accordingly, teacher education will have to care much more about the negotiation of the student teachers' subjective constructions of meaning The processual nature of such adaptations and personal reconstructions and their genesis through emergence rather than by organized elicitations require new ways of academical encounters, when these are to serve as a helpful experiential basis for the formation of wanted habitus characteristics. An experimenting with new forms of teacher education will be necessary: Preparing for better opportunities of the student teacher interactions *at the university level,* an experimenting with alternative presentations and curricula etc.

We also argue for a reformed role of teaching practice in the initial teacher education. The introductory encounters with the reality of the mathematics classroom will have to follow different aims and serve different purposes. Teaching practices during the initial teacher education should *not* have to serve for an introduction into the techniques of teaching and for developing professional skills (like micro-teaching efforts have tried e.g.). Rather the main purpose of such practice encounters should be seen in their function for

- the development of a sufficiently and personally experienced basis for the necessary theoretical reflexions and to raise the motivation for these,
- an opening of the student teachers' eyes for the hidden structures of the complex field, particularly of their own experiences as a school student (in retrospect) and the limitations arising from these, and thus for
- the furtherance of developing an alternative school mathematical habitus.

5. Options for Necessary Changes in Teacher Education

> "From the start, the child becomes readily attuned to "making a lot out of a little" by combination. He typically works on varying a small set of elements to create a larger range of possibilities." Bruner (1983, p. 29)

The following paragraphs are to describe more detailed recommendations for an innovation of the initial education of elementary mathematics teachers. (The order of the remarks does not present a hierarchy in any sense.)

5.1. The academic teaching of mathematics

Since in many countries faculties for mathematics have mathematics education as a subdepartment under their control, one can state an overestimation of function and importance of the pure mathematics parts in teacher education. But the need for complementing and reorienting

present teacher education is not so much with the syllabus of mathematical themes, their content and their extension, but with the ways of *how lecturers teach* it. The key deficit is with an organized theoretical grappling with the processes in the mathematical classroom. If one of the fundamental goals of elementary education is to help more children with developing a stable self-concept across experiences of doing things successfully, then, clearly, this is true with the academic education in mathematics as well. (In the average, in Germany, about 20% of the university students only who start studying mathematics pass a final examination in mathematics, not through failing the tests at the end but by giving up in between.)

Surely, we can not expect university lecturers in mathematics to act like elementary math teachers. But they will have to understand themselves as representatives of a culture not only of *doing* mathematics but also of *teaching and learning* mathematics. Through their activities the lecturers demonstrate their ways of mathematizing, of dealing with mistakes and incorrect inferences, of using mathematical models in other areas, and last not least of speaking about the issues, of using graphical and other means, and of representing outcomes, of varying tasks etc. In other words, the culture of academic mathematizing during the initial teacher education should not be too different from the culture of the later mathematical classroom, particularly it should not work contra-functional to the latter. This, to be clear, requires academic lecturers who *specialize* themselves in teacher training. (It is a well-known issue, though mostly treated as a neglectable one, that many university faculties all over the world hold their own and specialized lecturers for mathematics. So do physicists and chemists as well as psychologists or medical faculties. There is no consideration, surprisingly, on whether teacher students would need that desperately as well and from quite similar reasons.)

5.2. Fundamental Attitudes and the Investigations Approach

There is a simple chain of conditions from wanted activities on the student's side to the necessary prerequisites on the teacher's side. As we know, at school the students' mental development rests upon the quality of their repeated, varied, and engaged own activities. This quality, as we know as well, is intimately interrelated with the potential power and richness of the culture of the mathematical classroom (not at least with the teacher's inventions). And a crucial part of this culture is the teacher's permanent support of an *attitude of curiosity and of inquisitiveness* with the students, of searching for pattern and regularities, of expecting to find surprising issues, etc. Since the human nature is too complex and not tractable through simple cause-effect models, clearly, the chain is not reversible. But the teacher's part in this interaction forms a necessary, though not sufficient, condition for the development of the wanted activities: There is not much possibility for the students' developing and differentiating of such an attitude, if the described support is missing.

What is meant here is the *development of an attitude through participation* rather than "motivating" (as cognitive psychology has named it) or "making things interesting" through so-called applications or everyday relations. What is different in comparison with earlier "discovery" approaches? It is the shift from rare and inserted exceptional situations towards an integrated part of the life in the mathematics classroom:

Classical Discovery approach	**Integrated (Culture) approach**
- focus on the single learner -	*- focus on style in social interaction -*
In explicitly defined situations, the student 'researcher' starts off from an introduction to working on prepared material, and finally to a discussion and clearing of the findings class session.	*In every classroom situation, the students are expected to search for pattern, to assume regularities, and to relate developing or contrasting ideas, as well as to give reason and arguments for the issue under discussion.*

Aside of that there are fundamental doubts, which are not to be discussed here, about what the notion of "discovery" can describe at all from a constructivist perspective (see Bauersfeld, 1991).

The English "investigations" approach (see e.g. Mason, Burton & Stacey 1982, Burton 1984, Mottershead 1978 and 1985) and many inventions of the Freudenthal Institute in Utrecht/Netherlands (former IOWO) are promising realizations. These provide teachers with encouraging starting points: using open-ended tasks, exploiting momentary everyday situations, and providing for (relatively) selforganized processes of working on 'data', questions, and solutions (see the many contributions in the Dutch Journal "Willem Bartjens"; also de Lange 1987, Treffers 1991, and Streefland 1991). And the preferred organisation is small group work of pairs of students (dyadic interactions).

The initial teacher education will have to introduce the teacher student into such practices, since it is too late to expect later school practice to somehow automatically further such fundamental attitudes with the teacher. The reproductive power of the classical school system and of its ineffective minimal[3] solutions for everyday activities are too strong and too resistant against change. Why not organize seminars with the Brown/Walter book (1990), developing the art of actually varying problems and adapting such variations to different individual needs for

3 Indeed these are classical maximum-minimum solutions: they produce a maximum of smoothness with regular classroom processes requiring a minimum of effort on the teacher's side.

understanding and strategies? Why not organize selected teaching practices, where one teacher students works with two or three school students in related situations, - thus being exposed to a limited complexity of classroom reality only? Why not discuss and analyze concludingly the videotaped episode with both groups? etc.

5.3. Mathematical Language and Languaging

For many teachers the strength and the generalizability of mathematics is inseparable from the strictness and the precision of the related verbal or other symbolic representations. Similar to priests who celebrate the esoteric language game of their caste, many mathematics teachers permanently insist on saying and hearing issues formulated as 'sharp' as possible. An observer may find, the teacher insists on this technical language. For the students the force will function more in a direction of saying "it" exactly "as the teacher said it".

One may suspect whether many teachers do "have" their mathematical concepts at all in any other way. That is to say, they know how to *talk* about "it" in the terminology of the accepted language game. But there seems to be not much more beyond, as the limited availability in other 'contexts', the difficulties and shakiness with the use in other situations, and the inability of metaphorizing the issues adequately, indicate.

In order to be fair: As far as I have come to know nobody on the academic level encourages teachers in the initial phase to speak about the (mathematical) matter meant in everyday language, to "point at" similar issues etc. Cognitivists may prefer descriptions like: "translate", or "say it in other words", or "refer it to", and to "embed" or to "visualize", thus treating the matter meant as an object rather than as something emerging from the actually situated processes. In consequence, many mathematics teachers are quite rigid in their *verbal* aspirations and with their related evaluations of students' utterances. But they are quite permissive with the *social* organization of their class. From a social interactionist or constructivist perspective the other way round appears to be more promising: to accept and encourage students' mathematical utterances within very wide limits for the how-it-is-said and the metaphors used, as long as a serious background (reason, argument etc.) can be identified with it; but to be rigid in keeping the social regulations, insisting of listening to other's inventions and explanations, in keeping turn-taking order, in taking serious the others' serious contributions, etc.

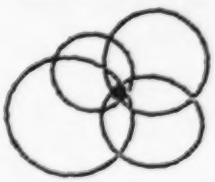
The analyses of many videotapes from different countries unveil the all too general poverty of classroom communication from this perspective. If the culture the students live in at their classroom is poor in languaging and in presenting models of the wanted, if it is lacking incentives and challenges, if it is more a non-transparent celebration of technical language rather than a participation in a scaffolding culture (see Bruner, 1983), and if it is neither providing resistance

for the critical mind nor further orientation for the keen minded, what then are we to expect from our schools and as a resulting school mathematical habitus?

The difficulty here is, not at least, with the negative effects of much of the initial *mathematical* instruction of teacher students. "Learning how to use language involves both learning the culture and learning how to express intentions in congruence with the culture" (Bruner & Haste, 1987, p. 89). And "one has to conclude that the subtle and systematic basis upon which linguistic reference itself rests must reflect a natural organization of mind, one into which we grow through experience rather than one we achieve by learning." (ibid., p. 88; emphasis in the original). This points at the need for a reasonable innovation of initial teacher education from the aspect of language and speaking, particularly related to the intimate interrelations between mother tongue, everyday language, figurative speech, and the technical (mathematical) language.

Benjamin Lee Whorf, mostly known because of the linguists' critics of his "Whorfian hypothesis" stating an intimate relation between thinking and language, has described a remarkable example of how precise natural language descriptions can be though these languages may use quite wide concepts (used with multiple meanings):

the intersection of of a few extensive concepts can be quite narrow and sharp, allowing many other connections



whilst the use of just one narrow concept can suffer from his poor relations to other (narrow) concepts

The paradox issue of this demonstration is with the presentation in mathematical set language.

Seminars on languaging mathematically, on "how to speak about the issue meant," on presenting given tasks with different words (and different graphical means), using adequate metaphors etc. may be useful (see Shuard & Rothary 1984). Aside of the expected developing of more empathy with the teacher student such experiences can provide for a more effective introduction into our knowledge of classroom communication and interaction.

5.4. Mistakes: Constructive Orientation versus Deficit Repair

> "People can help each other learn when they use their differences in a joint activity. Much schooling in America has a contrasting focus, namely, finding reasons why each others' contributions are inadequate."
>
> Bredo & McDermott
>
> (1992, p. 36; reviewing Newman, Griffin & Cole, 1989)

Since their professionally developed mathematical expectation functions as criterion, teachers, inevitably, perceive and assess a student's mathematically inadequate action as 'wrong'. Consequently - but didactically contrafunctional in most cases - their reaction aims at bringing the difference between the student's actual performance and the perfection of their own expected answer to zero.

As an example we can refer to the common didactical conviction that students' mathematical errors are mostly caused by a strategy or a rule. The usual attempt for repair then aims at replacing the faulty rule by the adequate one. But: If a student acts in a specific situation *as if* he followed a rule, this allows quite different interpretations:

a) The inadequate action can be taken as an indicator for a ready-developed but faulty strategy behind. In order to make the student correct her-/himself the perspective of deficit repair often leads the teacher to a continuous questioning, on and on, until, under growing tensions on both sides, the expected answer is heard, either from the student or in the end given by the teacher her/himself.

b) The inadequate action can also be taken as an indicator for the actual and tentative forming of an answer which appears acceptable in the present situation. Then the "missing" is understood as a *present* individual construction rather than as the product of something like conscious knowing, selecting, and applying of a rule. In this case there is nothing to "repair." It is more a case for the constructive *encouraging of appropriate parts* and steps in the faulty procedure, of developing these towards an acceptable solution, and passing over (neglecting) the weak parts rather than of *hardening the tentative construction* through an intensive discussion of the wrong parts and steps.

Students arrive at mental constructions and regularities with individually very different states of perfectness, fixedness, and openness to an actual shaping, according to the perceived situation. The latter strategy, therefore, appears to be more cautious in most situations. In the first case all too often the teacher leaves the episode with the impression of a successful invention and achieved repair. But because the ongoing questioning does not provide the student with a chance to directly adopt or "internalize" a relevant person's adequate rule, the teacher's impression turns into an illusion. Thus the teacher's "surgery" leaves the student with damaging effects onto his/her tentative constructing.

So, why not replace the *deficit criterion* by a *constructive orientation* more generally? Alternatively the teacher would have to identify the positive or convergent elements in the student's present producing and to change the actual task accordingly, so that the student gets a chance to extend his/her use of the positive elements (i.e. reinforce the positively working parts and connections). Teachers who try to adapt the actual tasks towards a possible extension of what

the student can do already more or less successfully (Vygotsky's "zone of proximal development") will surely elicit less frustration and more encouragement with their students than the usual forcing into contradictions and the continuously squeezing questioning does. Surely, such strategies, the identification of the adequate moment, and the helpful variations have to be learned and developed in action, - there is no simple verbal transmission.

Seen from a connectionist perspective (see Bauersfeld, 1993b), the offered alternative carries the potential for the emergence of 'better' (neuronal) connections through the achieving of accepted activities. Whereas the questioning method promotes the generation of weak connections, endangered by confusion and high emotional load. Interestingly, recent research work on the learning of orthography in language has arrived at the very same approach, replacing *instruction* about "mistakes" by the furtherance and guidance of individual *constructions* (see H. Bruegelmann's contribution in Lorenz, 1991).

If the student teacher does not learn her/his mathematics during the initial teacher education in a similar mode, how then will she/he change when entering the classroom? Again, we arrive at the need for lecturers in mathematics, who are specialized on educating teachers. The more the academic disciplines unfold and specialize themselves the more it will become necessary to have lecturers specializing in mathematics (and not only in mathematics education) for teacher students.

5.5. Taboos and their consequences

Teachers had been students before they became teachers. Thus teachers already have a well developed school mathematical habitus, when entering teacher education, though often not one we would find helpful. The initial teacher education, therefore, has to take into account deeply rooted orientations and options for the future teacher's actions rather than to build upon a tabula rasa fundamentum. One of the mostly unconscious parts are the taboos, things you should *not* do as a teacher. Classroom taboos are among the least discussed issues in mathematics education. But they belong to the most effective and covertly functioning forces in classroom realities. If teacher education does not touch these strong orientations and readily developed options for action the old non-reformed reality of school will just reproduce itself, irresistibly and reliably.

The "exhaustion method" of continuous questioning the student until the expected answer has been given is a consequence of the taboo: "Never tell a student what he can find by himself!" This taboo has been anchored as a guiding principle in many German syllabi, the state regulations for mathematics instruction at school. In-service teacher training institutions also disseminate this principle. It appears in the US as well. The damaging effect of the taboo is, that it all too often narrows into a contrafunctional "funnel pattern" of interaction (see Bauersfeld, 1978 and 1988).

Furthermore there is a clear relation to the suspected issue of "discovery" (see 5.2, and Bauersfeld, 1991).

Videotapes of typical classroom episodes will be helpful material for analyzing discussions with teacher students, - not only in this special case. If the tape gets stopped at crucial points in the classroom situation, the teacher students can try to answer questions like: "What do you think has gone wrong with the students thinking?" - "How would you, as the teacher, continue in this situation?" - "How to come, that just these ideas jump into your mind and no others?!" - "Are there at all different alternatives for adequate inventions?" - "Which principles, convictions, etc., do you think, guide the teacher's actions?" and so on. Through such analyses the teacher students may find out reflectively, which guiding regulations and orientations had formed their own activities as a school student, and, also, which ones alternatively may have guided the teacher's actions. And they can compare these outcomes with their own actual ideas and associations for their future activities as a teacher.

5.6. Geometry: Arithmetic's missing foundation

A key issue in the preparation of teachers for elementary mathematics is the missing part of geometry. Usually geometry plays a minor part in the Elementary math curriculum. And whenever time runs short at school, the geometrical themes are the first ones to be dropped. But: arrangements of counters, rods, and blocks; number strips and number line; dot cards, the arrays for multiplication, hundredsboard, addition and multiplication tables, dominoes, geoboard, representations for the concept of average, fractions as parts of circles and rectangulars, and so on, - all these presentations bear on geometrical structures. In the elementary textbooks there is no embodiment, no graphical representation for numbers and arithmetical operations which does not imply geometrical structures.

The tragedy is, that all these modeling devices are used in the classroom *as if* the child were in command of the relevant geometrical properties. As long as these devices seem to function, there is no reason to consider the underlying structures, as weak as the children's related construction of meaning may have been. And when the devices fail to serve, the teacher's correction is given usually from the arithmetical perspective. Thus, the horse is bridled from the backside: The students are forced to adapt their manipulations to the *arithmetical* results, instead of reversely using the *geometrical* properties for the fundamenting of the arithmetical operations and properties. Thus the function of embodiments is turned upside down.

Since school does not provide the teacher student with the necessary experiences, how then can we bring about change into the mathematics classroom? Clearly, lectures about abstract geometry will be of small help. Students as well as teachers do need exercises with materials and

geometrical models, cutting paper and cardboard, building solids. They also need rehearsals with mental operations on images of figures and solids; with moving, turning, and cutting the images and demonstrating the outcomes not only through language, but also through bodylanguage, hands and fingers (deictics). Useful are also exercises in how to speak about such operations and the changing properties, using everyday language and metaphors, - and not only the technical language of geometry experts. Clearly, we do not aim at mechanical drill here. Meant are the necessary explanations and reflections while working on the issues.

Once we do not prepare future teachers this way, the effects are easy to predict. The fast learners and children with positive learning dispositions will learn to come around anyway. The victims will be the disadvantaged children, - just those, who need support and "scaffolding" most of all, - for a public school system very contrafunctional effects in terms of negative social selections.

5.7. Teaching experiments in small groups

Many years ago Jack Easley has given convincing demonstrations of how useful teaching experiments with small groups of school students (see the related dissertations from Shirk, 1972; Driver, 1973; Erlwanger, 1975; also Bauersfeld, 1980). Related to seminars about the function of materials and embodiments, about students' learning, problem solving, etc., teacher students can teach small groups of school students. The advantage is an encounter with school practice in a situation of *reduced complexity,* accompanied by documentation through videotape. Repeated reviewing of the tapes, employing different analytical perspectives, follows subsequently.

Such teaching experiments are particularly useful, when the teacher students have had the chance before to extend their own experiences with the materials, their structure and their possible uses. Necessary themes (see 5.6.) for such rehearsals—due to the teacher students' deficits—are: solids and their nets and cuts, straw-models, plane figures and their properties, reflection and rotation, thumb movies, enlarging and reducing of figures, geoboard activities, Polyominoes, paperfolding and -cutting etc. All too many teachers have difficulties with organizing experiences for students, since they themselves did not have it, and, worse, because they consider it too difficult for the students. Once teachers take the risk, they often get aware of the fact, that children have much less difficulties with these tasks and can be faster. And many children do like these tasks.

Videotaping has allowed new approaches: classroom episodes can be selected and played back repeatedly, opening the chance for many different analyses of the very same episode from different perspectives and theoretical orientations. These documents are more reliable than field notes or reports from recollecting. Particularly a teacher, trying to remember parts of his/her own

lesson will, inevitably, mix experiences and intentions. But clearly, even the videotaping can provide for a relative objectivity only, because of the focus and the selections behind.

Again, the problem with these academic seminars and teaching experiments is the way of how they are run—or better: the culture rather than the materials or the mathematical themes (see Baruk 1985, and Brown & Walter 1990). How to make transparent the own thinking and constructing, but also disappointment and failure? How to understand what children do and try to demonstrate? How to vary tasks spontaneously, but adequate to the children's needs, and neither too early nor too late? How to encourage what is promising on the way, neither to invent too hasty nor to cut tentative action through too much criticism. The problem is *not the training* of these skills - they will develop across coherent practice only - but the related focus of attention, the awareness, and the preprarедness. These form the necessary prerequisites for every theoretical analysis and orientation.

6. In-service education

"By education most have been misled; so they believe, because they were so bred."
John Dryden (quoted in G. Bateson "Mind and Nature", Bantam, 1979, p. 26)

It is much easier to identify positive trends with recent in-service teacher education approaches. Countries like Britain and Norway have very early decentralized their institutions for the in-service support of teachers. Their very effectively working "teachers centres" have become wellknown and have served as exemplars for the development of similar institutions in many other countries. This came to happen at a time whilst in the US still many states organized their in-service support for schools in large regional Laboratories or Development Centers. Though there existed already different movements, teacher centers like those at SIU, Edwardsville, Ill. (Tom O'Brien) or in Chicago, Ill. (Sharon Feiman), et. al. (see review and evaluation of teachers centers for mathematics in Bauersfeld & Lange, 1979).

Common characteristics of all of these decentralized teacher centers approaches are

- the working groups, their goals, and their activities are defined by local teachers, also opening and termination of the working groups
- experts come in on invitation from teacher groups, but do not take over "courses" by themselves
- material development, curriculum ideas and other products serve the teachers' local needs and problems (copying or small scale printing facilities are available), but are not aiming at large scale distribution.

Across the last two decades in Germany the integration of teacher education into the universities has lead to a separation and an increasing independance of schools from the universities. Two models dominate in in-service education:

1) a *course* model organized by the school administration, offered to the teachers of a region. Participants get a few days leave from school; the staff of their school has to compensate their absence without additional help. A central institute for in-service and education[4], defines the themes, selects the participants, invites the experts, and pays for travel expenses and per diem. The Land, clearly, uses these courses to concretize and disseminate his curriculum regulations (which are in the authority of the Bundes-Länder).

2) a *workshop* model organized by groups of engaged teachers, "Lernwerkstatt". These institutions work without state support, in rooms near to a school. Private contributions from teachers enable the maintaining, also the hiring of one or two unemployed but experienced teachers. These workshops often organize after school help for low learners, in the afternoon, for the main subjects (math and reading/writing). They apply and try new materials and they offer workshops locally for interested teachers, e.g. each monday from 4-6 p.m. for three months (see the special edition of the periodical "Grundschule", Westermann Verlag, Braunschweig, 24, No. 6, June 1992, with adresses, typology, and case studies).

It is quite convincing to realize the dramatic shift away from model 1 to models and variations of type 2. Within a few years more than 80 of such free workshops have sprouted into an effective live. Obviously, the teachers win more self-confidence and want to organize help according to their definition of needs and problems. Certain of the Länder begin to react either by reducing the power of their central institute or even by dissolving their central institute and founding many smaller "satellite" institutions, like teacher centers, scattered over the country, which are to work more along locally defined difficulties.

The differences are at hand: the centralized state agencies used to produce tons of printed papers, circulated to all schools, but with very limited effect. They reached very, very few teachers directly, following an out-worn top-down philosophy of development, lecturing, and dissemination. The local workshops function reversely. They create changes through participation in interesting developmental work. Good materials or ideas are spread through the recommendations of teacher's, whence they worked successfully with the material or ideas. And good teachers find it quite rewarding to work in these institutions.

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4 With minor differences al of the (west) German Bundes-Länder run such central institutions, - still. Their tasks are mainly the preparation of the state regulations for the schools in all subject matters, the syllabi, and the training of teacher-teachers, or "multipliers", as they according to their expected function are called.

Two conditions I find crucial as prerequisites for changes in teaching, though I do not know any country where these have been realized:

1. Teachers should be allowed to visit each other in their classrooms - at least for one lesson per week - for to share teaching, and to learn from each other through direct cooperation.

2. The frustrating exploitation of outstanding teachers through overburdening them with additional tasks without receiving any equivalent for it should find an end. (In the workshops, e.g., I have never come to hear teachers complain about exploitation.)

7. Summing up

"... without the constituting role of culture we are unworkable monstrosities ..."
Bruner (1990, p. 12, quoting Clifford Geertz)

The core difficulty with most school reforms is their demanding of fundamental changes of teachers' attitudes, beliefs, and changes in the classroom culture. Once we begin to understand preservice teacher education, school reality, curriculum, administration, and in-service education as a *system*, in which the manipulation of a single factor will cause uncalculable disturbation and irritation rather than the expected change, things may begin to turn to the better. Perhaps, school reforms in future will become initiated and anchored at earlier stages and at more distributed points of invention, on a broader basis, and thus, in the end, may become more effective from the perspective of schools and students' mathematical competence. The reform of initial teacher education, surely, will form a core part of such endeavours.

The radical constructivist principle, that every cognitive construction is a person's individual construction, leads to the inference that the students at least will have to find themselves in a situation which allows and inspires constructions and personal activities, and more so: constructions which potentially go beyond the already available. It is the concept of **culture**, which I prefer for these descriptions. What we find in a mathematics classroom, described more precisely, is a kind of subculture, "a matter less of artifacts and propositions, rules, schematic programs, or beliefs, than of associative chains and images that tell what can be reasonably linked up with what." (Michelle Rosaldo, quoted by Bruner in Bruner & Haste, 1987, p. 90). If the education of mathematics teachers is to become more effective, it will be necessary to organize the related academic culture less contra-functional and nearer to the desired culture of the mathematics classroom.

Adopting Luria's notion of "romantic science" the closure at least has to be made with stories. Therefore, two stories follow, related to concrete classrom experiences:

- Many years ago Nancy Shelley from Australia has written a thesis about her work in mathematics with a few second graders in Neuguinea, who were dropouts from school. She worked intensively with them in analytic geometry on parabolas in particular. She taught these students to cup both their hands like a parabola according to her oral presentation of an equation like "y equals three x-square" or "y equals a half x-square" etc. The students liked these exercises, particularly when she jumped from equation to equation and the students had to adapt their hands adequately.

  Once Nancy opened such rehearsal with a student, starting with "y equals x-square".He reacted as expected, cupping his hands to form a normal parabola.Then she said "y equals two x-square." He did not react. She repeated the formula with a slight stress on the "2" and looked into his eyes. Thus every student in the world, experienced across many mathematics lessons, would know from that that something is wrong with his use of the factor 2. But this boy--eye to eye--did not react. Nancy again repeated the formula, raised in tone now. And he responded: "I changed the scale!"
- A case from my class[5]: In first grade, the third month about, the teacher made the children show numbers with their fingers: "Show me 4!" The children raise four fingers. "Now, can you show me 3, using *two* hands?" The children used one hand for stretching out 1 finger of one hand and 2 fingers of the other hand. "Can you show me 5 with *three* hands!" The kids laughed, a few said: "I got two hands only!" But a few took their neighbours hand, made him stretch 1 finger, and raised their own hands each with 2 fingers stretched out. Then the teacher tried to tease them: "Can you show *one* using two hands?" Again they laughed and seemed to be helpless. One little bright-eyed girl raised both hands with the thumbs up, - bent over.

Mostly, I think, such cases are treated under headlines like "creative production" as nice, but as a stroke of luck, of windfall, as unguidable like a butterfly and thus seen as to be different from routines, skills, and everyday operations. But once we identify the human interactions in the mathematics classroom with a subculture, helpful questions arise: To which extent do our usual approaches to operations and skills in classrooms contribute to the development of such events? To which extent does teacher education prepare future teachers to realize, to accept, and to further such events? Isn't it the case that every action contributes to the culture, reinforcing routines as well as the butterfly productions? How then can we hope to create a culture - at schools as well as at universities - which in all relevant dimensions will support serious and productive mathematizing?

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5 Since August 1988 I have shared teaching an elementary class in mathematics with an experienced woman teacher. We started with the first day in first grade. The study ended July 1992 with the end of grade four. We have changed many details in curriculum, particularly on geometry, as well as in the social organization.

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